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Simulations of Astrophysical Fluid Instabilities

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Abstract. We present direct numerical simulations of mixing at Rayleigh-Taylor unstable interfaces performed with the FLASH code, developed at the ASCI/Alliances Center for Astrophysical Thermonuclear Flashes at the University of Chicago. We present initial results of single-mode studies in two and three dimensions. Our results indicate that three-dimensional instabilities grow significantly faster than two-dimensional instabilities and that grid resolution can have a significant effect on instability growth rates. We also find that unphysical diffusive mixing occurs at the fluid interface, particularly in poorly resolved simulations.

INTRODUCTION

Many of the problems of interest in relativistic astrophysics involve fluid instabilities. The shock of a core-collapse supernova propagating through the outer layers of the collapsing star, for example, is subject to Rayleigh-Taylor instabilities occurring at the boundaries of the layers. A fluid interface is said to be Rayleigh-Taylor unstable if either the system is accelerated in a direction perpendicular to the interface such that the acceleration opposes the density gradient or if the pressure gradient opposes the density gradient [1,2]. Growth of these instabilities can lead to mixing of the layers. The early observation of ^{56}Co, an element formed in the core, in SN 1987A strongly suggested that mixing did indeed play a fundamental role in the dynamics. Following this observation, supernova modelers embraced multi-dimensional models with the goal of understanding the role of fluid instabilities in the core collapse supernova process [3]. Despite years of modeling these

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events, many fundamental questions remain concerning fluid instabilities and mixing. In this manuscript, we present early results of our research into resolving fluid instabilities with FLASH, our simulation code for astrophysical reactive flows.

The FLASH code [4] is an adaptive mesh, parallel simulation code for studying multi-dimensional compressible reactive flows in astrophysical environments. It uses a customized version of the PARAMESH library [5] to manage a block-structured adaptive grid, placing resolution elements only where needed in order to track flow features. FLASH solves the compressible Euler equations by an explicit, directionally split version of the piecewise-parabolic method [6] and allows for general equations of state using the method of Colella & Glaz [7]. FLASH solves a separate advection equation for the partial density of each chemical or nuclear species as required for reactive flows. The code does not explicitly track interfaces between fluids, so a small amount of numerical mixing can be expected during the course of a calculation. FLASH is implemented in Fortran 90 and uses the Message-Passing Interface library to achieve portability. Further details concerning the algorithms used in the code, the structure of the code, verification tests, and performance may be found in Fryxell *et al.* [4] and Calder *et al.* [8].

RESULTS

From our single-mode Rayleigh-Taylor studies, we find significantly faster instability growth rates in three-dimensional simulations than in two-dimensional simulations. In addition, we find that obtaining a converged growth rate requires at least 25 grid points per wavelength of the perturbation, that grid noise seeds small scale structure, and that the amount of small scale structure increases with resolution due to the lack of a physical dissipation mechanism (such as a viscosity). Another result is that poorly-resolved simulations exhibit a significant unphysical diffusive mixing. Figure 1 shows the growth of bubble and spike amplitudes for two well-resolved simulations beginning from equivalent initial conditions. The three-dimensional result (left panel) shows faster growth than the two-dimensional result (right panel). Results of our single-mode studies will appear in Calder *et al.* [9].

Our single-mode studies serve as a prelude to multi-mode studies, which are works in progress; our single-mode results strongly suggest that using sufficient resolution is essential in order to obtain physically-sensible results for these calculations. In the multi-mode case, bubble and spike mergers are thought to lead to an instability growth according to a t^2 scaling law, which for the case of a dense fluid over a lighter fluid in a gravitational field may be written as [10]

$$h_{b,s} = \alpha_{b,s} g A t^2 \tag{1}$$

where $h_{b,s}$ is the height of a bubble or spike, g is the acceleration due to gravity, $A = (\rho_2 - \rho_1)/(\rho_2 + \rho_1)$ is the Atwood number where $\rho_{1,2}$ is the density of the lighter (heavier) fluid, and t is the time. α is a proportionality 'constant' that may be thought of as a measure of the efficiency of potential energy release. Experiments

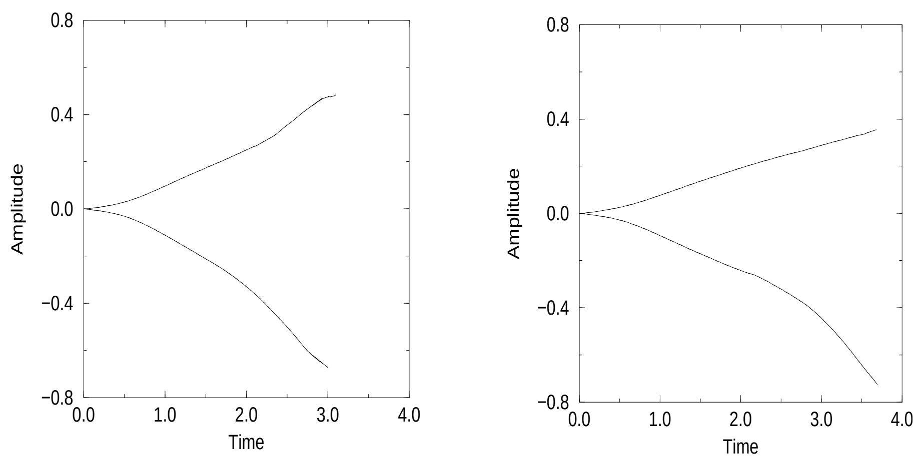

FIGURE 1. Bubble and spike amplitudes for two-dimensional (right) and three-dimensional (left) simulations of single-mode instabilities. The resolutions are 128 X 768 (2-d) and 128 X 128 X 768 (3-d). The amplitudes are measured by tracking the advection of each fluid. The initial conditions consisted of a dense fluid ($\rho = 2$) over a lighter fluid ($\rho = 1$) and $g = 1$. The initial perturbation consisted of a sinusoidal vertical velocity perturbation of 2.5% of the local sound speed with the horizontal components chosen so the initial velocity field was divergence-free.

and simulations indicate that α lies in the range of 0.03 to 0.06, and it is thought to depend on Atwood number, evolution time, initial conditions, and dimensionality. See Young *et al.* [11] and references therein for a discussion of experimental results. Results of our multi-mode studies will appear in publications of the Alpha Group, a consortium formed by Guy Dimonte in 1998 to determine if the t^2 scaling law holds for the growth of the Rayleigh-Taylor instability mixing layer, and if so, to determine the value of α [12].

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